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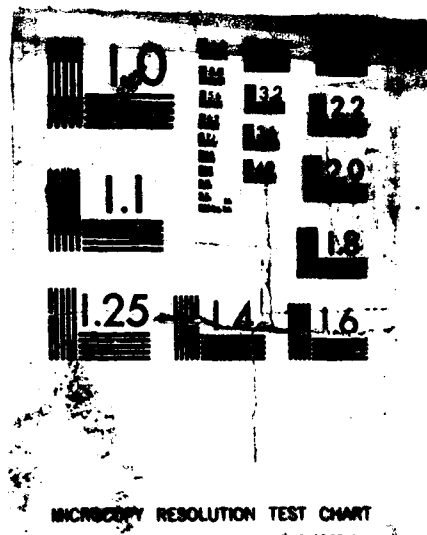
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TECHNICAL REPORT BRL-TR-2795

AN APPLICATION OF THE FINITE  
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TOMOGRAPHIC IMAGE RECONSTRUCTION

R. T. SMITH  
C. K. ZOLTANI

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## I. INTRODUCTION

We examine here a method for reconstructing an x-ray attenuation function from measurements of its integrals in the case where the projection data is sparse. In practice, data from as few as 5 to 20 projections, with perhaps only 25 measurements per projection, may be all that is available. This is the case, for example, in industrial nondestructive testing, where the object whose density cross-section we are attempting to determine is in a rapid state of flux. In this event, there is very little time available in which to gather data. Further, what little data there is available is almost certainly degraded by noise and possibly blurring due to the motion of the object.

One should note that the standard reconstruction algorithms currently in use in most medical CAT scanners utilize 180 views (or projection angles) and yield very poor resolution when the number of views is substantially reduced<sup>1</sup>. One approach which has proven useful when the available projection data is sparse is the maximum entropy algorithm MENT of Mänerbo<sup>2</sup>. In this method, one attempts to compute the attenuation function having maximum entropy (intuitively, one might say, the function yielding the picture with the least information content<sup>2,3</sup>) and which is also consistent with the measurements. One of the difficulties with this approach is that Mänerbo looks for a solution in the class of functions continuous in the scanning region, while it has been shown by Klaus and Smith<sup>4</sup> that the solution of this problem must be piecewise constant throughout the scanning region. Also, the MENT algorithm as now implemented does not take advantage of a priori information known about the object being scanned. We address both of these problems in the current work. The most significant difference, however between MENT and the current work is that MENT requires that the density of the object being scanned tends to zero as the edges of the scanning region are approached. If this requirement is not met, MENT may produce a completely unrecognisable image. Naturally, the only way to guarantee that such an "edge condition" is satisfied in practice, is to require minimum x-ray source-to-phantom and phantom-to-detector distances, where the density of the surrounding medium is known a priori. This condition, then imposes a restriction on the maximum size of an object to be scanned by a given device, a size smaller than that which the machinery itself would otherwise allow. The current method removes this restriction entirely. We have included an example of a reconstructed density profile with nonzero density near the edge of the scanning region, obtained by the new method. As a comparison, we include the MENT reconstruction obtained using the same data.

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<sup>1</sup>C.K. Zoltani, K.J. White and R.P. Kruger, "Result of Feasibility Study on Computer Assisted Tomography for Ballistic Applications", ARBRL-TR-02513, Aberdeen Proving Ground, Maryland, ADA 133 214, August, 1983.

<sup>2</sup>G. Mänerbo, "MENT: A Maximum Entropy Algorithm for Reconstructing a Source from Projection Data", *Comp. Graph. and Image Proc.*, Vol. 10, pp.48-68, 1979.

<sup>3</sup>R.D. Levine and M. Tribus, The Maximum Entropy Formalism, MIT Press, Cambridge, 1978.

<sup>4</sup>M. Klaus and R.T. Smith, "A Hilbert Space Approach to Maximum Entropy Reconstruction", *Math. Meth. in the Appl. Sci.*, to appear.



In [4], Klaus and Smith demonstrated that by subtracting the entropy from a penalty term consisting of the residual error in matching the measurements and then minimising this augmented cost functional over a weakly compact set, one obtains a well-posed optimisation problem in  $L^2_+(D)$ , where  $D$  is the scanning region and  $L^2_+(D)$  is the space of non-negative square integrable functions on  $D$ . That is, there exists a unique solution of the problem and the solution depends continuously (weakly in  $L^2_+(D)$ ) on the measured data. It was also shown that the solution of this problem, for the special case of the weakly compact set which we consider here, must be piecewise constant. Our problem is set in the larger function class  $L^2(D)$ , rather than in the class of piecewise constant functions, however, since the geometry of the sets on which the solution is constant is so complicated as to make trying to find the exact solution of the optimization problem prohibitive.

In the present work, a solution of the optimization problem is sought over a finite dimensional subspace of  $L^2(D)$ . Thus, the infinite-dimensional optimization problem is reduced to one over  $R^n$ . A simple discretization of  $D$  is made and a space of finite elements is defined on this grid. We restrict ourselves here to basis functions which are piecewise constant or products of piecewise linear functions in each variable. It is shown here that as the finite element mesh size tends to zero, the sequence of solutions of the finite dimensional optimization problems will converge, in  $L^2(D)$  to the solution of the original, infinite dimensional problem.

The finite dimensional optimisation problems are somewhat large in practice (usually at least 300 to 1,000 variables, for minimal resolution) but they are relatively simple in form. Although the cost functional is quite nonlinear, the constraints are very simple bound constraints.

For our numerical experiments, we have used the MINOS nonlinear optimization package of Stanford's Systems Optimization Laboratory<sup>5,6</sup>. The current version of MINOS uses a reduced gradient scheme together with a quasi-Newton method.

## II. THE OPTIMIZATION PROBLEM

We consider the problem of reconstructing an x-ray attenuation function in two dimensions from measurements of its integrals. The x-rays are assumed to be collimated so that a parallel beam scanning geometry is obtained. Assume that  $J$  x-ray sources

<sup>5</sup>P. Gill, W. Murray and M. Wright, Practical Optimization, Academic Press, London, 1981.

<sup>6</sup>B.A. Murtaugh and M.A. Saunders, "Large-Scale Linearly Constrained Optimization", Math. Prog., Vol. 14, pp.41-72, 1978.

are arranged on a circle centered at the origin and containing the region to be scanned,  $D \subset \mathbb{R}^2$ . Suppose that the center lines of the x-ray projections make angles  $\alpha_j$ ,  $j = 1, \dots, J$ , with the positive x-axis. We assume that we have available measurements of the x-ray attenuation along strips  $A_{jm}$ ,  $m = 1, \dots, M(j)$ , which are parallel to the center line of the x-ray projection from the  $j^{\text{th}}$  source, for each  $j = 1, \dots, J$ . The strips are such that for each  $j = 1, \dots, J$ ,

$$D \subset \bigcup_{m=1}^{M(j)} A_{jm}.$$

The entropy of  $f$ , for  $f \in L^2(D)$ , is defined to be

$$\eta(f) = - \int \int_D |f(x,y)| \ln [ |f(x,y)| A ] \, dx \, dy, \quad (2.1)$$

where  $A$  is the area of  $D$ .

Let  $f_0$  be the actual x-ray attenuation function. We will assume that there is measured projection data  $G_{jm}$  available in the form

$$G_{jm} = S_{jm}(f_0) = \int \int_{A_{jm}} f_0(x,y) \, dx \, dy, \quad (2.2)$$

$m = 1, \dots, M(j); j = 1, \dots, J.$

We add to the entropy a penalty term corresponding to the residual error in meeting the measurements (2.2). Define the functional

$$G(f) = -\eta(f) + \gamma \sum_{j,m} [G_{jm} - S_{jm}(f)]^2, \quad (2.3)$$

where  $\gamma > 0$  is the penalty parameter.

The object then, is to minimize (2.3), subject to some appropriate constraints on  $f$ . This should yield the picture with the least information content which matches the measured data to within a specified error. [Note that the residual error is reduced by taking

larger values of the penalty parameter  $\gamma^7$ .] We then consider the optimization problem

$$\inf_{f \in \Sigma} G(f), \quad (2.4)$$

where  $\Sigma$  is a weakly (sequentially) compact subset of  $L^2_+(D)$ . By weakly (sequentially) compact, we mean that every sequence in  $\Sigma$  has a weakly convergent subsequence whose weak limit lies in  $\Sigma$ . Of course,  $\{x_n\}$  weakly convergent to  $x$  means that the sequence of real numbers  $\{(x_n, y)\}$  converges to  $(x, y)$  for every  $y$  in  $L^2(D)$ , where  $(\cdot, \cdot)$  denotes the usual  $L^2$  inner product. In practice,  $\Sigma$  will contain a priori information known about the object being scanned (e.g.  $\Sigma$  may specify maximum and minimum densities, information on the support of the attenuation function, etc.).

In [4], it was shown that the optimization problem (2.4) is well-posed. That is, there exists a unique solution for any given set of measured data and the solution depends continuously (weakly in  $L^2(D)$ ) on the measured data. The key to the proof of well-posedness hangs on the following two results, which will also be needed to demonstrate the convergence of our numerical method.

Lemma 1:

The cost functional  $G$  is continuous on  $L^2(D)$ .

Lemma 2:

$G$  is a strictly convex, weakly lower semicontinuous functional on  $L^2_+(D)$ . By weakly lower semicontinuous, we mean that if  $\{f_n\}$  converges weakly to  $f \in L^2_+(D)$ , then

$$G(f) \leq \liminf_{n \rightarrow \infty} G(f_n).$$

For the proof of Lemmas 1 and 2, the interested reader is referred to [4].

It is also shown in [4], that the solution of (2.4) for the special case of  $\Sigma$  considered here is constant on each cell of  $D$ , where by a cell, one means the interior of a maximal region which is not crossed by any of the strip boundaries. This characterization of the solution, while of theoretical interest, is of little help in actually solving the optimization problem, because of the very complicated geometry involved. Rather than using this characterization of the solution, then, we consider a method of solution utilizing the  $L^2(D)$  structure.

<sup>7</sup>R. Fletcher, Practical Methods of Optimization, Vol. 2, John Wiley, New York, 1980.

One should note that, due to the severe nonlinearity of the object function  $G(f)$ , it is rather difficult to approach (2.4) as an infinite dimensional problem using the calculus of variations. We therefore approach the problem via the Ritz method<sup>8</sup>. For each  $h > 0$ , let  $S^h$  denote a finite dimensional subspace of  $L^2(D)$ , satisfying the following approximation property : Given  $\epsilon > 0$ , one can choose  $h$  sufficiently small so that the best approximation  $f^h$  to  $f$  out of  $S^h$  (i.e. the projection of  $f$  onto  $S^h$ ) satisfies

$$\| f - f^h \|_{L^2(D)} < \epsilon. \quad (2.5)$$

Consider now solving the optimization problem (2.4) over  $S^h$ , that is,

$$\inf_{f \in \Sigma \cap S^h} G(f), \quad (2.6)$$

for a sequence of values of  $h$ , tending to zero. We can now prove the following convergence result.

Theorem:

Let  $\{ S^h \mid h > 0 \}$  be a family of finite dimensional subspaces of  $L^2(D)$  satisfying the above approximation property. Then for

$$\mu_h^* = G(f_h^*) = \inf_{f \in \Sigma \cap S^h} G(f),$$

$\mu_h^*$  converges to  $\mu^*$  and  $f_h^*$  converges weakly ( in  $L^2(D)$  ) to  $f^*$ , as  $h$  tends to 0, where

$$\mu^* = G(f^*) = \inf_{f \in \Sigma} G(f).$$

Proof:

Since  $G$  is weakly lower semicontinuous and strictly convex on  $L^2_+(D)$  and  $\Sigma$  is weakly compact, there must exist a unique minimizer,  $f_h^*$  of  $G$  over  $\Sigma \cap S^h$ , for each  $h > 0$ . Let  $f^h$  denote the best approximation to  $f^*$  out of  $S^h$ . Since  $G$  is continuous on  $L^2(D)$  (Lemma 1) and the family of subspaces  $S^h$  satisfies the above approximation property, one may, given  $\epsilon > 0$ , choose  $h > 0$  sufficiently small so that

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<sup>8</sup>I.M. Gelfand and S.V. Fomin, Calculus of Variations, Prentice Hall, Englewood Cliffs, New Jersey, 1963.

$$G(f^h) < G(f^*) + \epsilon. \quad (2.7)$$

Note that

$$\mu^* = G(f^*) = \inf_{f \in \Sigma} G(f) \leq \inf_{f \in \Sigma \cap S^h} G(f) = G(f_h^*). \quad (2.8)$$

However, since  $f^h \in S^h$ ,

$$G(f_h^*) = \inf_{f \in \Sigma \cap S^h} G(f) \leq G(f^h).$$

From (2.7) and (2.8), it then follows that

$$\mu^* \leq G(f_h^*) \leq G(f^h) < G(f^*) + \epsilon = \mu^* + \epsilon,$$

and hence,  $G(f_h^*)$  converges to  $\mu^*$ , as  $h$  tends to 0. Note that this alone is not sufficient to say that  $f_h^*$  converges to  $f^*$  in  $L^2_+(D)$ . Since one is interested in approximating the attenuation function and not just its associated entropy, this last question is crucial. This can be resolved as follows.

Let  $\{h_n\}$  be a sequence of positive real numbers tending to 0 and let

$$\mu_n = G(f_n^*) = \inf_{f \in \Sigma \cap S^{h_n}} G(f).$$

Then  $\{f_n^*\}$  is a minimizing sequence for  $G(f)$  over  $\Sigma$ , from the preceding. Since  $\Sigma$  is weakly (sequentially) compact,  $\{f_n^*\}$  must have a weakly convergent subsequence  $\{f_{n_k}^*\}$  whose weak limit  $f^{**}$  lies in  $\Sigma$ . It follows by the weak lower semicontinuity of  $G$  that

$$G(f^{**}) \leq \lim_{k \rightarrow \infty} G(f_{n_k}^*) = \mu^* = G(f^*),$$

whence  $f^{**} = f^*$ , since  $G$  has a unique minimizer in  $\Sigma$ . Lastly, since  $\Sigma$  is weakly compact, and  $\{f_n^*\} \subset \Sigma$ , every subsequence of  $\{f_n^*\}$  has, in turn, a subsequence converging weakly to  $f^*$ . Thus,  $\{f_n^*\}$  must itself converge weakly to  $f^*$ .

One can see from the theorem that, by solving the optimization problem (2.6) for a finite dimensional subspace  $S^h$  of  $L^2(D)$ , we do indeed obtain an approximation to the solution of the infinite dimensional problem (2.4). In the next section, we give examples of two finite element spaces in which we have solved (2.6). We have included examples of images reconstructed with this method using extremely sparse data.

### III. FINITE ELEMENT APPROXIMATION.

In this section, we show how to implement the method described in the last section by using a space of finite elements as the approximating subspace of  $L^2(D)$ . We give several examples of simple finite element spaces and reconstruct several images using one of these spaces.

First, superimpose a fixed rectangular mesh on  $D = [-1,1] \times [-1,1]$ , with uniform mesh size,  $h = 1/n$  in both the  $x$  and  $y$  directions, for simplicity. Consider using products of piecewise linear functions in  $x$  and  $y$  as the finite element space  $S^h$ . A basis for  $S^h$  is given by

$$\phi_k(x,y) = \phi_i(x) \phi_l(y), \quad k = 1, \dots, (2n+1)^2,$$

where

$$\begin{aligned} l &= [(k-1) - (k-1) \bmod (2n+1)] / (2n+1) - n, \\ i &= k - (l+n)(2n+1) - n - 1 \end{aligned}$$

and

$$\phi_j(t) = \begin{cases} 0 & , \quad t \leq (j-1)h, \quad t \geq (j+1)h \\ [t - (j-1)h] / h & , \quad (j-1)h \leq t \leq jh \\ [(j+1)h - t] / h & , \quad jh \leq t \leq (j+1)h. \end{cases}$$

It is very reasonable to expect that in practice, one should know a priori the minimum and maximum densities of the object being examined. Therefore, we have defined a simple constraint set by

$$\Sigma = \{f \in L^2(D) \mid 0 < a \leq f \leq b < \infty, \text{ a.e. and } f=0 \text{ a.e. in } \mathbb{R}^2 \setminus D\}$$

The attenuation function  $f$  is then approximated in  $S^h$  by

$$f(x,y) = \sum_{k=1}^N c_k \phi_k(x,y),$$

where  $N = (2n+1)^2$  and the coefficients  $c_k$  are chosen as the solution of the finite dimensional optimization problem

$$\inf_{\underline{c} \in \mathbb{R}^N} G \left( \sum_{k=1}^N c_k \phi_k(x,y) \right) \quad (3.1)$$

subject to

$$0 < a \leq \sum_{k=1}^N c_k \phi_k(x,y) \leq b. \quad (3.2)$$

With the simple choice of the finite element space  $S^h$  and due to the form of  $G$ , (3.1) - (3.2) reduces to

$$\inf_{\underline{c} \in \mathbb{R}^N} \left\{ -\eta \left( \sum_{k=1}^N c_k \phi_k(x,y) \right) + \gamma \sum_{j,m} [G_{jm} - \sum_{k=1}^N c_k S_{jm}(\phi_k(x,y))]^2 \right\} \quad (3.3)$$

subject to

$$0 < a \leq c_k \leq b, \quad k=1,2,\dots,N. \quad (3.4)$$

The finite dimensional optimization problem (3.3) - (3.4) can be solved numerically, for sufficiently small values of  $N$ , using existing optimization software packages. [For our examples, we have used the MINOS package<sup>6</sup>.] Note that this can be done only if one has a simple method of computing values of the object function for each updated value of  $c_1, \dots, c_N$ . Because of the form of (3.3), one needs to compute the values of  $S_{jm}(\phi_k(x,y))$  for each  $j, m$  and  $k$  only once for given projection and discretization geometries. These values can then be stored in a (somewhat large) disk file and simply read back each time new values of  $c_1, \dots, c_N$  are computed (i.e. in each pass of the optimization scheme). We have devised an algorithm which can compute the integrals  $S_{jm}(\phi_k(x,y))$  exactly for any given projection and discretization geometries. It is then a simple matter to compute the residual error

$$\sum_{j,m} [G_{jm} - \sum_{k=1}^N c_k S_{jm}(\phi_k(x,y))]^2, \quad (3.5)$$

at each pass of the optimization scheme.

Note that due to the nonlinearity of the entropy term, we need to recompute

$$\eta \left( \sum_{k=1}^N c_k \phi_k(x,y) \right)$$

for each new set of values  $c_1, \dots, c_N$ . Although the geometry is simple, the integrands are rather complicated. One can write the entropy as an iterated integral, the first of which is evaluated explicitly. The second integration is then performed numerically, each time a new set of values of  $c_1, \dots, c_N$  is computed. In the examples, we have used an adaptive quadrature routine for this.

The question of how to determine an optimal value of the penalty parameter  $\gamma$  is as yet unresolved. In practice,  $\gamma$  must be sufficiently large so that the residual error in meeting the measurements (3.5) is small. At the same time,  $\gamma$  must not be so large that the penalty term completely swamps out the entropy. Computational experience has been our only guide thus far, for determining an acceptable value of  $\gamma$ . More work remains to be done in this direction.

As yet, nothing specific has been said about how to choose the mesh size  $h$ . In the theorem of the last section, we have seen that as  $h$  tends to 0, the solution of (3.3) - (3.4) tends to that of the infinite dimensional problem (2.4). In practice, however, one must choose a value of  $h$  which will yield a reasonable resolution and yet for which the computer time is not excessive. There is of course, also an important restriction placed on  $h$  by our desire to be able to choose  $\gamma$  sufficiently large so that the residual error (3.5) is within tolerances. In order that one might make (3.5) arbitrarily small, it is necessary that the mesh be sufficiently fine that the system of equations

$$G_{jm} = \sum_{k=1}^N c_k S_{jm}(\phi_k(x,y)), \quad m = 1, \dots, M(j); \quad j = 1, \dots, J$$

is consistent (i.e. there must be at least one solution). For the examples which we have considered, we have screened prospective values of the mesh size by numerically checking the system of measurement equations for consistency. We then increased the value of the penalty parameter until a minimizer with an acceptable residual error was found.

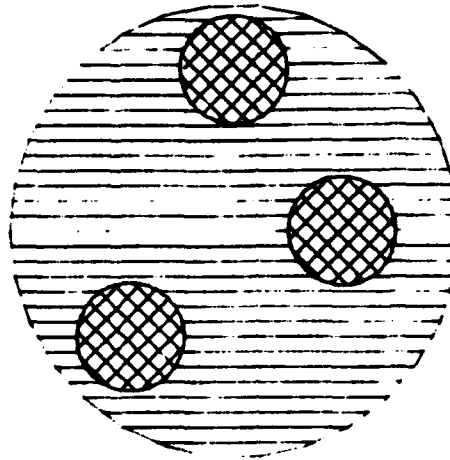
As an illustration of the reconstruction algorithm presented above, we consider a phantom consisting of three identical cylinders, all of density 1, placed at arbitrary positions within the scanning region, where the background density in the scanning region is .1 (see Figure 1 for a cross-sectional view). Note that one of the cylinders is rather close to the edge of the target grid. For the reconstruction, the phantom is set within a square grid 30 unit cells on a side.

The computer code GOLEM<sup>9</sup> was used to produce the x-ray absorption data from the phantom, mathematically. Data was produced from 25 detectors for each of 5 projection angles, arranged uniformly around the target. The reconstruction grid spans the width of the 25 detectors, as seen in Figure 1. The 31x31 grid is used as the finite element grid and in Figures 2 and 3 can be seen inside a 51x51 grid, where points outside the 31x31 grid are given density zero.

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<sup>9</sup>MD. Altschuler, T. Chang and A. Chu, "Rapid Computer Generation of 3-D Phantoms and Their Cone-Beam X-ray Projections", Medical Image Processing Group Technical Report MIPG16, State University of New York at Buffalo, Nov. 1978.





#### SAMPLE PROBLEM

	XC	YC	RAD	DENSITY
BACKGROUND	0.0	0.0	15.0	0.1
1	-7.0	-7.0	3.6	1.0
2	0.0	10.7	3.6	1.0
3	7.2	0.0	3.6	1.0

Figure 1. Cross-Sectional View of Phantom

In Figure 2, we give the reconstructed density plot, using our new method, for the phantom in Figure 1. Notice that the three cylinders stand out clearly from the background. In Figure 3, we give the MENT reconstruction using the same data as that used for Figure 2. Of course, this is completely unrecognizable, due to the large mass near the edge of the target grid.

A more extensive collection of examples is given in [10]. More of the technical details of the reconstruction are given there, along with a more detailed comparison with MENT.

#### ACKNOWLEDGEMENT

The authors gratefully acknowledge the assistance of Dr. George Klem of the Ballistic Research Laboratory for his assistance in obtaining projection data and for porting the codes to the Cray XMP/12 computer.

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<sup>10</sup>R.T. Smith, C.K. Zoltani and G. Klem, "Reconstruction of Tomographic Images from Sparse Data by a New Finite Element Maximum Entropy Approach", BRL Report, in preparation.

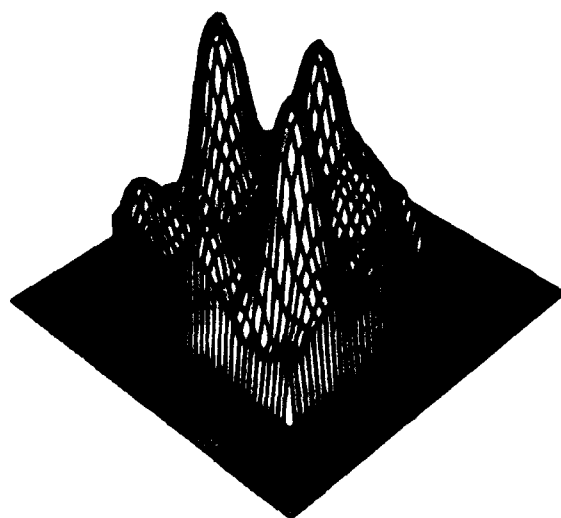


Figure 2. Reconstructed Density Plot using New Method

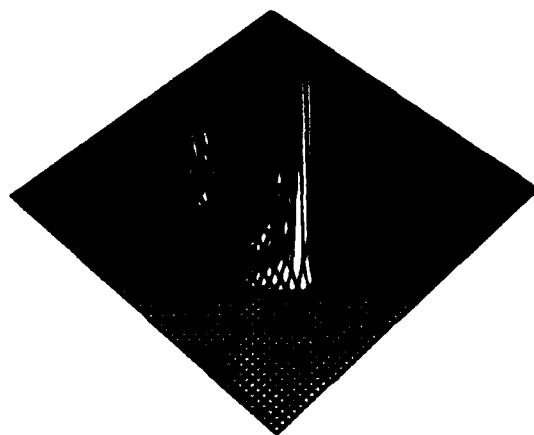


Figure 3. Reconstructed Density Plot Using MENT

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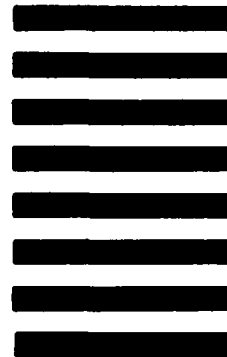


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